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The effect of the local inertial term on the fluid flow in channels partially filled with porous material

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Abstract

The transient hydrodynamics behavior of the fluid flow in parallel-plate channels partially filled with porous material is investigated numerically. The role of the local macroscopic inertial term in the porous domain momentum equation is studied. It is found that the effect of the local inertial term on the channel hydrodynamics behavior is insignificant when $Da < 1 \times 10^{-6}$, over the entire range of $0.1 < \mu_R < 10$, $0 < A < 10^4$, and for all porous substrate thicknesses. Also, it is found that the deviation between the predictions of the transient and the quasi-steady models is more significant in the porous domain and the deviation decreases as the time proceeds.

Keywords: Porous media; Transient; Two-phase

1. Introduction

Fluid flow problems in closed conduits, partly filled with porous media, have received great attention recently. This attention is motivated by several engineering applications of this specific area. These applications include: porous journal bearing, nuclear reactor, geothermal systems, solid matrix heat exchangers, porous flat plate collectors, thermal insulation, storage of nuclear waste materials, grain storage and drying, and many others [1].

The literature shows that several investigators have studied the steady-state characteristics of the hydrodynamics as well as the heat transfer behavior of flows through closed conduits partly filled with porous material. The steady hydrodynamics behavior of the fluid flow in channels partly filled with porous material is first investigated by Beavers and Joseph [2] who present an empirically based correlation for the velocity gradient at the clear fluid/porous interface in terms of the velocities in the fluid layer and the porous region. The same problem is solved analytically using the matched asymptotic expansion technique by Vafai and Thiyagaraja [3] and solved exactly by Vafai and Kim [4]. The transient hydrodynamic behavior of the fluid flow in channels partly filled with porous material is investigated by Al-Nimr and Alkam [5], where the unsteadiness in the hydrodynamic behavior is due to a step change in the imposed pressure gradient. The pulsating flow in channels and tubes totally filled with porous material is investigated analytically by Alkam and Al-Nimr [6]. The thermal behavior of flow through domains partly filled with porous material is investigated by many researchers [7–15]. In the present work, the transient hydrodynamics of a fluid flow inside parallel-plate channels partly filled with porous material is investigated numerically. The unsteadiness in the fluid flow is due to sudden change in the imposed pressure gradient which drives the flow.

The main goal of the present study is to investigate the role of the macroscopic local inertial term in the porous domain momentum equation and its effect on the hydrodynamics behavior of channels partly filled with porous material. In the literature about fluid flow in domains totally filled with porous material, it has been realized that the local macroscopic inertial term is usually small compared to the microscopic Darcy drag term, and hence can be neglected [16]. In most practical situations, the velocity responds to an imposed pressure

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Nomenclature

A	dimensionless Forchheimer coefficient $[(-\mathrm{d}p/\mathrm{d}x)(Fh_2^4\rho_1/\mu_1^2\sqrt{K})]$
c_a	acceleration coefficient tensor
Da	Darcy number, K_2/h_2^2
h_1	clear domain width
h_2	channel width
K	permeability of the porous medium
p	pressure
R	dimensionless ratio, h_1/h_2
t	time
t_0	reference time, $\rho_1 h_2^2/\mu_1$
u	axial velocity
u_0	reference axial velocity $[(-\mathrm{d}p/\mathrm{d}x)(h_2^2/\mu_1)]$
U	dimensionless axial velocity, u/u_0
x	axial coordinate
y	transverse coordinate
Y	dimensionless transverse coordinate, y/h_2

Greek symbols

μ	dynamic viscosity
μ_R	dynamic viscosity ratio, μ_2/μ_1
ρ	density
τ	dimensionless time, t/t_0

Subscripts

1	refers to the clear domain
2	refers to the porous domain

change within a second or less. The local inertial term may be important if an oscillatory pressure gradient is imposed or if the porous domain is of large void fraction. However, it is obvious that the local inertial term may retain its importance in applications involving very thin porous substrates or at large Darcy number. A quantitative mapping of the operating and geometrical parameters within which the local inertial term may be significant is not available in the literature yet. Also, the effect of these geometrical and operating conditions on the steady-state time, need to be investigated.

In this study, the Darcy–Brinkman–Forchheimer model is adopted to describe the fluid flow hydrodynamic behavior. The tangential velocities and stresses are assumed to be matched at the clear fluid/porous domains interface. The inclusion of the Brinkman term is justified when the porous domain is thin, i.e., $\epsilon > 0.6$ [17]. The continuity of the tangential velocities and shear stresses at the interface is widely used in the literature. It is also believed that this approach gives good predictions especially in thin porous domains when $\epsilon > 0.6$ [1].

2. Mathematical formulation

Consider an unsteady laminar fully developed forced fluid flow into a parallel-plate channel partly filled with a porous material. The unsteadiness in the fluid flow is due to a sudden change in the pressure gradient which drives the flow. The fluid is assumed to be Newtonian with uniform properties and the porous medium is isotropic and homogeneous. Refering to Fig. 1, and using the dimensionless parameters given in the nomenclature, the equations of motion in both clear and porous domains are given as, respectively,

$$\frac{\partial U_1}{\partial \tau} = 1 + \frac{\partial^2 U_1}{\partial Y^2}, \tag{1}$$

$$c_{\mathrm{a}}\frac{\partial U_2}{\partial \tau} = 1 + \mu_R\frac{\partial^2 U_2}{\partial Y^2} - \frac{1}{Da}U_2 - AU_2^2. \tag{2}$$

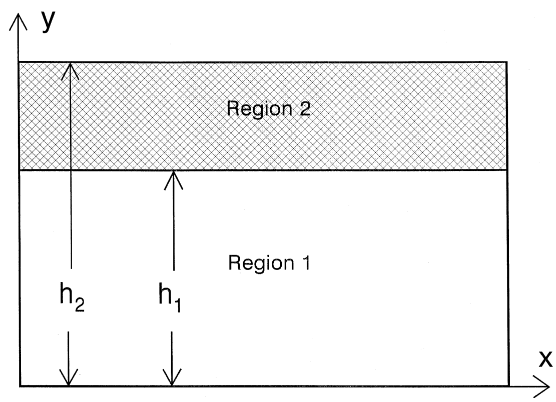

Fig. 1. Schematic of the problem showing the physical parameters.

In Eqs. (1) and (2), subscripts 1 and 2 refer to the clear fluid and porous substrate, respectively. In Eq. (2), c_{a} is an acceleration coefficient tensor that depends on the geometry of the porous medium [16]. The value of this coefficient is far from being settled and it may assumed to be $1/\epsilon$ or 1 in thin domains having void fractions close to 1. The other parameters appearing in Eqs. (1) and (2) are defined as

$$\mu_R = \frac{\mu_2}{\mu_1}, \qquad Da = \frac{K}{h_2^2}, \qquad A = \frac{-\mathrm{d}p}{\mathrm{d}x}\frac{Fh_2^4\rho_1}{\mu_1^2\sqrt{K}}.$$

Eqs. (1) and (2) are subject to the following initial and boundary conditions:

$$U_1(0,Y) = U_2(0,Y) = 0 \tag{3}$$

$$U_1(\tau,0) = U_2(\tau,1) = 0 \tag{4}$$

$$U_1(\tau,R) = U_2(\tau,R) \tag{5}$$

$$\frac{\partial U_1(\tau,R)}{\partial Y} = \mu_R\frac{\partial U_2(\tau,R)}{\partial Y} \tag{6}$$

3. Results and discussion

The non-dimensional system of coupled partial differential equations is solved numerically using the finite difference technique on a uniform grid. Second order accurate central differencing, except at the interface, is used for the spatial coordinate. At the interface, second order backward and forward differencing schemes are used in regions 1 and 2, respectively. The fully implicit scheme is used for the time derivative. Although the implicit scheme is only first order accurate in time, the use of the second order accurate Crank–Nicolson scheme imposes severe restrictions on the time step required for a bounded solution [18]. The problem rises from the term that included the Darcy number in Eq. (2). The requirement for a bounded solution necessitates the use of a time step at least an order of magnitude less than the lowest Darcy number used, 10^{-14} in this study. This will result in extremely long computational time before reaching steady-state conditions. The fully implicit method is not subject to such a restriction. Thus, a more practical time step can be used leading to considerable saving in computational time. Newton linearization technique is used for the last term in Eq. (2) [19]. The resulting system of linear algebraic equations is tri-diagonal except at the interface where the equation is in terms of five unknown node values. This is a direct result of using second order accurate forward and backward differencing schemes at the interface. The number of unknowns is reduced to three by addition and subtraction of the equation with those equations for the nodes just above and just below the interface. This makes the whole system tri-diagonal. The system is then solved using the Thomas tri-diagonal matrix algorithm [18,19]. The solution is advanced in small time steps until the change in the velocity at all nodes becomes less than a pre-defined steady-state tolerance value. At this point the solution is deemed to have reached steady-state.

Extensive testing is carried out for the proper values of the number of nodes, time step, and steady-state tolerance. This is important to ensure that the results were independent of such choices. The tests included varying the value of nodes (m) from 50 to 200, value of time step ($\mathrm{d}\tau$) from 10^{-4} to 10^{-8}, and the value of the steady-state tolerance (δ_t) from 10^{-8} to 10^{-12}. The tests are performed for several combinations of R, μ_R, Da, and A. The combination of parameters used in this study are $m = 100$, $\mathrm{d}\tau = 10^{-6}$, and $\delta_t = 10^{-10}$.

To verify the validity of the adopted numerical scheme, the steady-state versions of Eqs. (1)–(6), with negligible microscopic inertial term ($A = 0$), are solved analytically. The analytical solution under these assumptions is given as

$$U_1 = -\frac{Y^2}{2} + C_1 Y, \tag{7}$$

$$U_2 = Da + C_2 \sinh \lambda Y + C_3 \cosh \lambda Y, \tag{8}$$

where

$$C_1 = \frac{\Delta_1}{\Delta}, \qquad C_2 = \frac{\Delta_2}{\Delta}, \qquad C_3 = \frac{\Delta_3}{\Delta},$$

and

$$\begin{aligned}
\Delta &= R[\mu_R\lambda \sinh \lambda R \sinh \lambda - \mu_R\lambda \cosh \lambda R \cosh \lambda] \\
&\quad - [\cosh \lambda R \sinh \lambda - \sinh \lambda R \cosh \lambda], \\
\Delta_1 &= -Da[\mu_R\lambda \sinh^2 \lambda R - \mu_R\lambda \cosh^2 \lambda R] \\
&\quad + \left(\frac{1}{2}R^2 + Da\right) \\
&\quad \times [\mu_R\lambda \sinh \lambda R \sinh \lambda - \mu_R\lambda \cosh \lambda R \cosh \lambda] \\
&\quad - R[\cosh \lambda R \sinh \lambda - \sinh \lambda R \cosh \lambda], \\
\Delta_2 &= R[-Da\mu_R\lambda \sinh \lambda R + R \cosh \lambda] \\
&\quad - \left[-Da \cosh \lambda R + \left(\frac{1}{2}R^2 + Da\right) \cosh \lambda\right], \\
\Delta_3 &= R[-R \sinh \lambda + Da\mu_R\lambda \cosh \lambda R] \\
&\quad - \left[-\left(\frac{1}{2}R^2 + Da\right) \sinh \lambda + Da \sinh \lambda R\right].
\end{aligned}$$

Fig. 2 shows a comparison between the steady-state numerical and the analytical velocity profiles for $R = 0.5$, $\mu_R = 1$, $Da = 0.01$ and $A = 0$. As clear from this figure, the results are in excellent agreement.

In the following discussion, the model that excludes the local inertial term from the porous domain momentum equation will be referred to as the quasi-steady (Q-S) model while the model that includes the local inertial term will be referred to as the transient (T) model.

Fig. 3 presents a mapping for the regions within which the local inertial term in the porous domain momentum equation is insignificant. From this figure, the local inertial term is insignificant when $Da < 10^{-6}$, over the entire range of $0.1 < \mu_R < 10$ and for all porous substrate thicknesses (1-R). However, for substrates of thicknesses larger than 0.1, the local inertial term becomes insignificant when $Da < 10^{-4}$ and over the range of $0.1 < \mu_R < 0.6$. For $0.6 < \mu_R < 10$, one has to ensure that $Da < 10^{-6}$ in order to exclude the local inertial term in channels having substrates of thicknesses larger than 0.1. When the substrate thickness is larger than 0.25, one may exclude the local inertial term if $Da < 10^{-4}$ and over the range of $0.1 < \mu_R < 5$. It is worth mentioning here that the microscopic inertial term AU_2^2 does not affect the quantitative or the qualitative behavior of the previous mapping over the entire range included in this study, $0 < A < 10^4$.

Fig. 4 shows the effect of Da, A, R and μ_R on the needed time to reach steady-state behavior (τ_{ss}). This

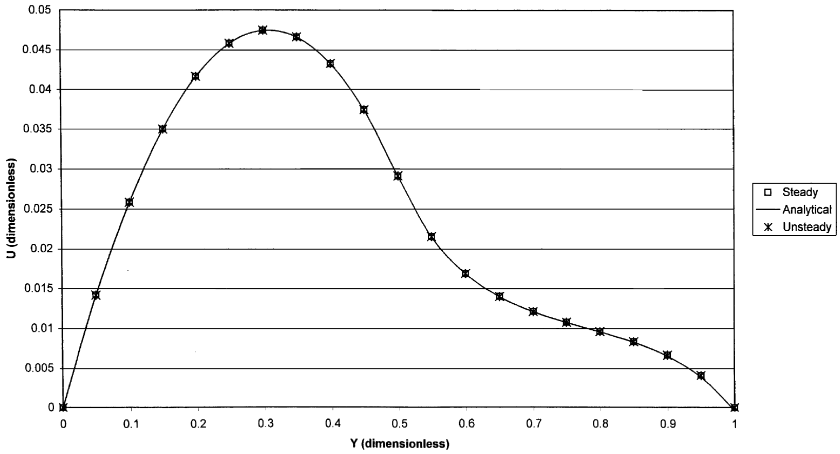

Fig. 2. Comparison between the analytical, quasi-steady, and transient steady-state velocity profiles for the case: $R = 0.5$, $\mu_R = 1.0$, $Da = 0.01$, and $A = 0.0$.

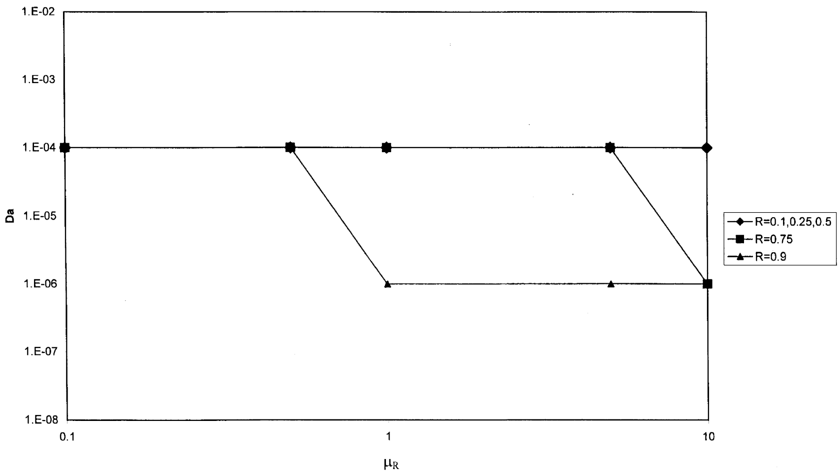

Fig. 3. Combinations of R, μ_R, and Da below which the local inertial term becomes insignificant.

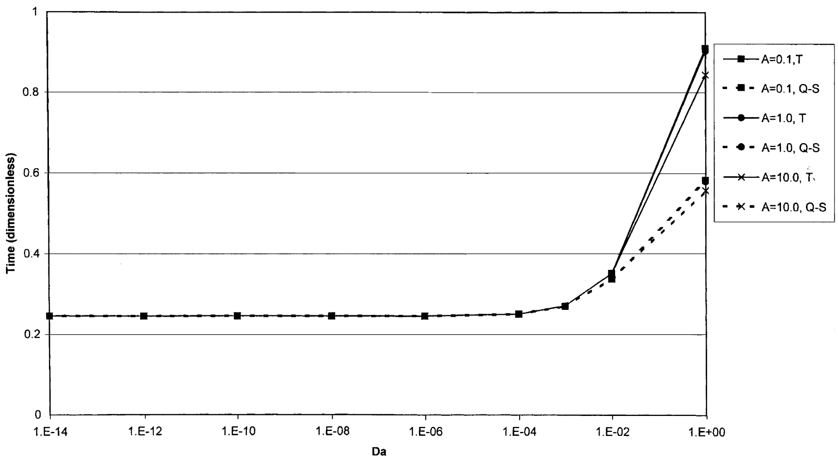

Fig. 4. Comparison between the non-dimensional time required to reach steady-state for the transient (T) and quasi-steady (Q-S) solutions vs Darcy number at different values of A. $\mu_R = 1.0$ and $R = 0.5$.

time measures the duration of the transient behavior and gives a clear indication about the importance of this period. If this time is very small, then the unsteadiness in the hydrodynamic behavior may be ignored regardless of the deviation between the predictions of the transient and the quasi-steady models during this short period. It is clear from this figure that for $Da < 10^{-4}$, the effect of A on τ_{ss} is insignificant and both the transient and the quasi-steady models give almost identical predictions. This agrees with our previous conclusions drawn from the mapping in Fig. 3. Fig. 4 shows that the deviation between the results of the two models decreases as A increases because the weight of the local inertial term with respect to the microscopic inertial term decreases as A increases. Also, it is clear from Fig. 4 that as A increases the steady-state time τ_{ss} decreases. As A increases, the retarding forces caused by the solid matrix increases and the net amplitude of the driving forces, which is the net effect of the pumping pressure gradient minus the microscopic inertial and viscous forces, decreases. This reduction in the driving forces causes a reduction in the mean velocity in the channel. As a result, the time required for the fluid to be accelerated from rest to this low mean velocity decreases. This is the

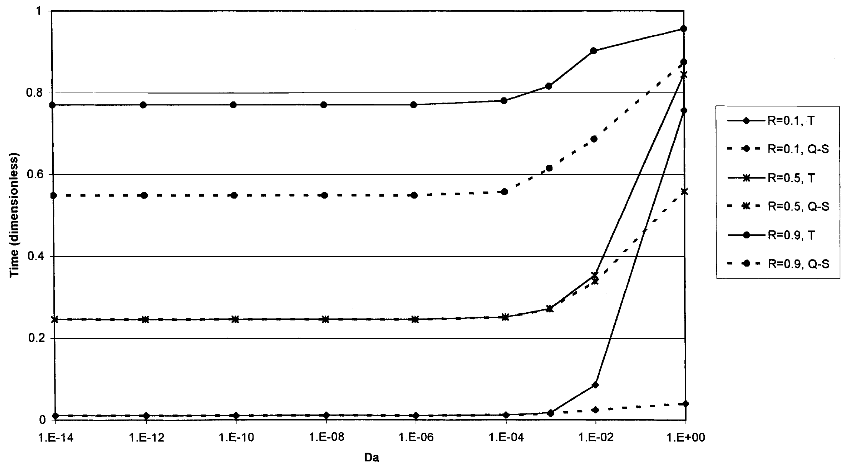

Fig. 5. Comparison between the non-dimensional time required to reach steady-state for the transient (T) and quasi-steady (Q-S) solution vs Darcy number at different values of R. $\mu_R = 1.0$ and $A = 10.0$.

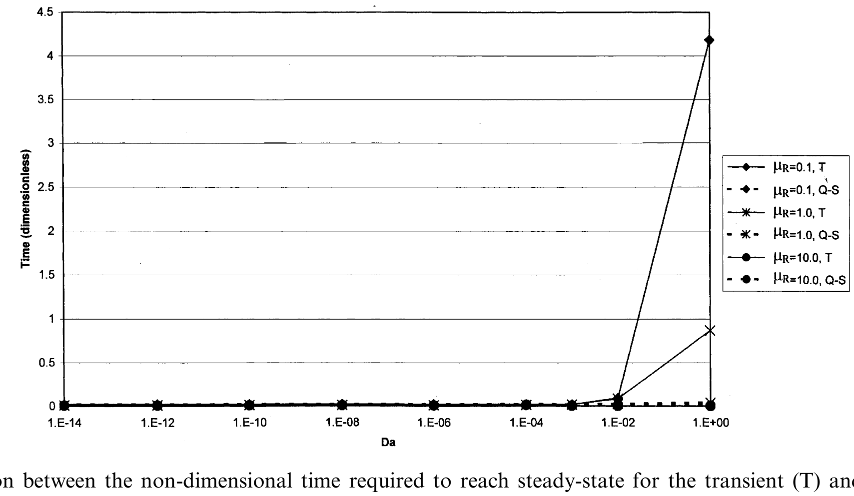

Fig. 6. Comparison between the non-dimensional time required to reach steady-state for the transient (T) and quasi-steady (Q-S) solution vs Darcy number at different values of μ_R. $R = 0.1$ and $A = 10.0$.

same reason why as Da decreases, τ_{ss} decreases. The coefficient of the microscopic viscous term plays the same role as of A.

Fig. 5 shows the effect of Da on τ_{ss} at different R. For $Da < 10^{-4}$, the effect of Da on τ_{ss} is insignificant for all substrate thicknesses (1-R). As the porous substrate thickness increases, the steady-state time decreases and the deviation between the predictions of the transient and quasi-steady models decreases. Also, it is clear that a slight decrease in the thickness of a thin substrate leads to a significant increase in τ_{ss}, while a slight decrease in the thickness of a thick substrate leads to an insignificant increase in τ_{ss}. In channels totally filled with porous material, the fluid responds almost immediately to the imposed pressure gradient. However, this is not the case when the porous substrate is thin. As an example, consider water flow in 0.1 m width channel, when the porous substrate thickness is 0.09 m ($R = 0.1$), the fluid attains its steady behavior after about 100 s ($\tau_{ss} = 0.01$). On the other hand, the fluid needs about 8000 s ($\tau_{ss} = 0.8$) to attain its steady behavior when the substrate thickness is 0.01 m ($R = 0.9$).

Fig. 6 shows the effect of Da on τ_{ss} at different μ_R. It is clear that μ_R has minimal effect on τ_{ss} for $Da < 10^{-2}$. Also, as μ_R increases, the steady-state time decreases especially at large Da numbers. Increasing μ_R leads to an increase in both the microscopic and the macroscopic

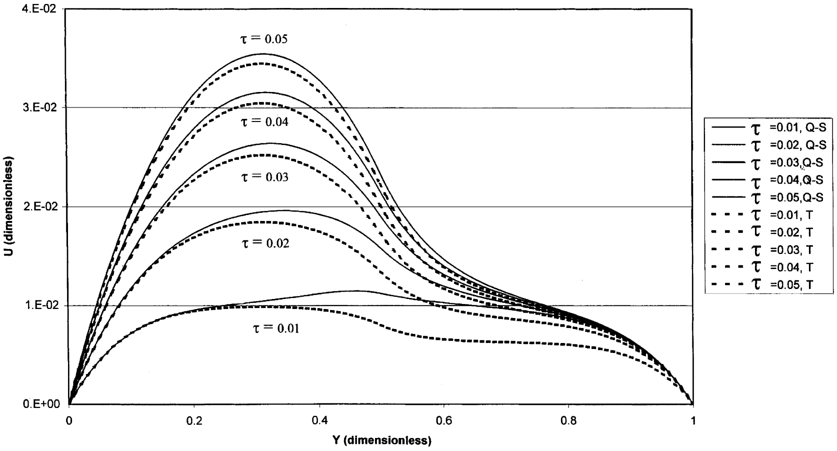

Fig. 7. Transient (T) and quasi-steady (Q-S) velocity profiles at selective non-dimensional times. $R = 0.5$, $\mu_R = 1.0$, $A = 10.0$, and $Da = 0.01$.

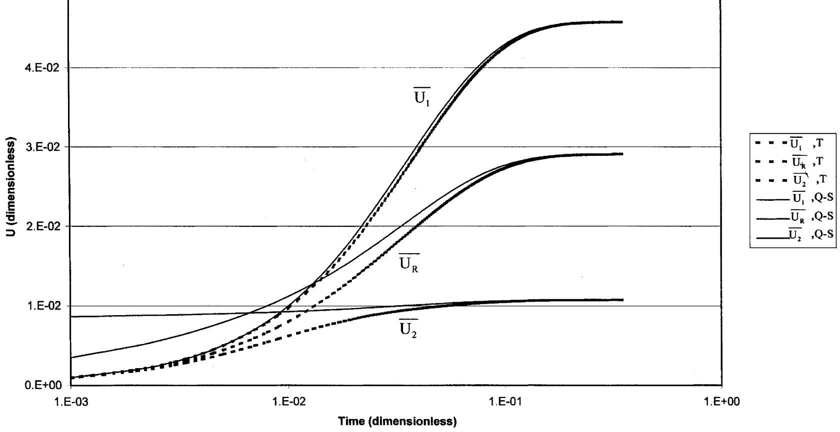

Fig. 8. Temporal development of the transient (T) and quasi-steady (Q-S) velocities at the center of the clear domain ($\overline{U1}$), at the interface (R), and at the center of the porous region ($\overline{U2}$). $R = 0.5$, $\mu_R = 1.0$, $A = 10.0$, and $Da = 0.01$.

retarding forces which reduces the mean velocity of the fluid flow in the channel. As a result, the time required for the fluid to be accelerated from rest to reach this low velocity decreases.

The transient and quasi-steady velocity profiles are shown in Fig. 7 at selective time steps. It is obvious that the deviation between the predictions of the two models is more significant in the porous domain. This implies that neglecting the local inertial term from the porous domain momentum equation does not alter the real hydrodynamic behavior of the fluid in the clear domain. Thus, if one is interested in estimating the local shear stresses at the wall adjacent to the clear domain, the quasi-steady model gives good predictions in this case. Also, it is clear that the deviation decreases as time proceeds and in the limit as $\tau \to \infty$, both models give exactly the same predictions.

Fig. 8 shows the temporal velocity development at three different locations: the center of the clear region, the center of the porous region, and at the interface between the porous and the clear domains. It is clear from this figure that the deviation between the predictions of the two models decreases as the time increases and the deviation in the porous domain is larger than that in the clear domain.

4. Conclusion

Numerical solutions are obtained for the transient fluid flow problem in channels partially filled with porous material under the effect of sudden change in the imposed pressure gradient. The effect of the porous medium local inertial term is investigated. A mapping is presented for the regions within which the local inertial term in the porous domain momentum equation is insignificant. It is found that the local inertial term is insignificant when $Da < 10^{-6}$, over the entire range of $0.1 < \mu_R < 10$ and for all porous substrate thicknesses. However, for substrates of thickness larger than 0.1, the local inertial term becomes insignificant when $Da < 10^{-4}$ and over the range of $0.1 < \mu_R < 0.6$. For $0.6 < \mu_R < 10$, one has to ensure that $Da < 10^{-6}$ in order to exclude the local inertial term in channels having substrates of thickness larger than 0.1. When the substrate thickness is larger than 0.25, one may exclude the local inertial term if $Da < 10^{-4}$ and over the range of $0.1 < \mu_R < 5$. Also, it is found that the microscopic inertial term does not affect the quantitative or the qualitative behavior of the previous mapping for $0 < A < 10^4$. The effect of different parameters on the steady-state time is also investigated. It is found that at small Da numbers, the parameters A, R, μ_R and Da have insignificant effect on the steady-state time. The deviation between the predictions of the transient and the quasi-steady models is more significant in the porous domain.

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